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Some properties of left regular semigroups satisfying the Identity xyz = xz

P. Sreenivasulu Reddy[a] & Gosa Gadisa[a]
Department of mathematics, Samara University Samara, Afar Region, Ethiopia. Post Box No.132

Abstract:- In this paper author proved that a left regular semi group S satisfying the identity xyz = xz for x, y, z in S is cancellative. It is also proved that S is a completely regular, commutiviy, left (right) quasi-normal, left (right) semi-normal and left (right) semi-regular. The motivation to prove this theorems in this paper is due to the results of Yamada, M. and Kimura, N.[3]

Keywords:- Cancellative, Left rugular Semi group, Quasi-normal semi group,

I. INTRODUCTION

Various concepts of regularity have been investigated by R.Croisot[10] and his study has been presented in the book of A.H.Clifford and G.B.Preston[1] as croisot's theory. One of the central places in this theory is held by left regularity. Bogdanovic and Ciric in their paper, "A note on left regular semigroups" considered some aspects of decomposition of left regular semigroups. In 1998, left regular ordered semigroups and left regular partially ordered Γ-semigroups were studied by Lee and Jung, kwan and Lee. In 2005, Mitrovic gave a characterization determining when every regular element of a semigroup is left regular. In this paper we discuss the structures of left regular semigroups which satisfies the identity xyz = xz for x, y, z in S. The results obtained in this section based on the results of Yamada, M. and Kimura, N. [3]. For definitions refer [3],[4],[5] and [6].

II. PRELIMANARIES

Theorem2.1. Let S be a left regular semigroup. Let x,y,z be the (choosen) elements of S. Then S satisfies the identity xyz = xz and $(xyz)^2 = xyz$

Proof. Let S be a left regular semigroup. Choose x,y,z in S such that z = zxz, x = xyzx, y = yzy. Since S is a left regular semigroup, we have
xyz = xy(z) = xy(zxz) = (xyzx)z = xz. Therefore, S satisfies the identity xyz = xz
To prove xyz is an idempotent element of S, consider $xyz = (xyzx)yz = (xyz)(xyz) = (xyz)^2$

Theorem2.2. Let S be a left regular semigroup satisfying the identity xyz = xz for some x,y,z in S. Then S is left and right quasi-normal.

Proof. Let S be a left regular semigroup. Then by Theorem 2.1,
$xyz = (xyz)^2 \Rightarrow xyz = (xyz)^2 = xyzxyz = xy(zxy)z = xyzyz = (xyz)yz = xzyz \Rightarrow xyz = xzyz$. Hence S is left quasi-normal
Again $xyz = (xyz)^2 = xyzxyz = xy(zxy)z = xyzyz = xyzxzyz = xyxzyz = xyxyz = xyxz \Rightarrow xyz = xyxz \Rightarrow$ S is right quasi-normal
Therefore, a left regular semigroup is left and right quasi-normal

Theorem2.3. Let S be a right regular semigroup satisfying the identity xyz = xz for some x,y,z in S. Then S is left (right) quasi-normal semigroup.

Proof: Proof is similar to Theorem 2.2.

Theorem2.4. Let S be a left(right) regular semigroup satisfying the identity xyz = xz for some x,y,z in S. Then S is a normal semigroup.

Proof. Let S be a left regular semigroup, then by Theorem2.1. for any $x, y, z \in S$, $xyz = (xyz)^2 \Rightarrow xyz = (xyz)(xyz) = xy(zxy)z = xy(zy)z = (xyz)yz = xzyz \Rightarrow xyzx = xzyzx \Rightarrow = xz(yzx) \Rightarrow xyzx = xzyx$
Therefore, S is normal.

Theorem2.5. Let S be a left(right) regular semigroup satisfying the identity xyz = xz for some x,y,z in S. Then S is right(left) semi-normal.

Proof. Let S be a left regular semigroup and x,y,z,∈S.Then $xyz = (xyz)^2$.
Now $xyz = (xyz)^2 = xyzxyz = xy(zxy)z = xyzyz = xyzxzyz = xyxzyz = xyxyz \Rightarrow xyz = xyxz \Rightarrow xyzx = xyxzx$. Hence S is right semi-normal.

Theorem2.6. Let S be a left (right) regular semigroup satisfying the identity xyz = xz for some x,y,z in S. Then S is left and right semi-regular semigroup.

Proof. Let S be a left regular semigroup and x,y,z,∈S.Then $xyz = (xyz)^2$.
$xyz = (xyz)^2 = xyzxyz = xy(zxy)z = xyzyz = xyzxzyz = xyxzyz = xyxyz \Rightarrow xyz = xyxz \Rightarrow xyzx = xyxzx = yx(zx) = xyx(zyx) = xyx(zy)x = xyxzxyx = xyxzx(yx) \Rightarrow xyzx = xyxzxyzx$. Hence S is a left semi-regular semigroup
Similarly $xyz = (xyz)^2 = xyzx(yz) = xyzxyxz \Rightarrow xyzx = xyzxyxzx$
Therefore, S is a right semi-regular semigroup.

Theorem2.7. A left regular semigroup S satisfying the identity xyz = xz for some x,y,z in S. Then S is cancellative.

Proof. Let S be a left regular semigroup and x,y,z are elements of S
Let xy =xz. Then $xzy = xyz \Rightarrow xzyzy = xyzxz \Rightarrow xzyzy = xyzxyz \Rightarrow xzyzy = xyzyz \Rightarrow xyzy = xzyz \Rightarrow xzxy = xzyxz \Rightarrow xyz(xy)^2 = xzy(xz)^2 \Rightarrow xyzx^2y^2 = xzyx^2z^2 \Rightarrow xyxy^2 = xzxz^2 \Rightarrow xyy = xzz \Rightarrow xy^2 = xz^2 \Rightarrow y = z$. Hence, S is left cancellative.
Similarlly we prove that S is right cancellative. Threrefore, S is cancellative.

Theorem2.8. Let S be a left regular semigroup satisfying the identity xyz = xz for some x,y,z in S. Then S is completely regular.

Proof. Let S be a left regular semigroup. Then by Theorem.3.3.7., S is cancellative
Let a be an element of S, then there exists an element x in S such that $xa^2 = a \Rightarrow xa^2x = ax \Rightarrow xaax = ax \Rightarrow xaax = (ax)^2 \Rightarrow xaax = axax \Rightarrow xaa = axa \Rightarrow xa = ax. \rightarrow (1)$. Also $xa^2 = a \Rightarrow x(xa^2) = xa \Rightarrow (xa)^2 = xa \Rightarrow xaxa = xa \Rightarrow axa = a \Rightarrow$ a is regular. By (1) ax = xa. So, S is completely regular.

Theorem2.9. Let S be a left regular semigroup satisfying the identity xyz = xz and a, b are elements of S. Then ab and ba are left regular elements in S.

Proof. Let S be a left regular semigroup. If a,b∈S then there exists x,y in S such that axa = a and byb = b. To prove that ab is left egular element in S, consider $yx(ab)^2 = yxa^2b^2 = y(xa^2)b^2 = yab^2 = a(yb^2) = ab$. Therefore, ab is a left regular element in S.
Similarly, we see that ba is a left regular element in S.

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